THE INFLUENCE OF RANDOM PROCESSES OF POWER REDUCTION ON THE MOTION OF A BODY OF VARIABLE MASS IN A GRAVITATIONAL FIELD

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The optimization of parameters for reactive motion of a body of variable mass in a gravitational field with constant jet power $N \equiv$ const as well as the case of multistep reduction of power source weight G_N with the corresponding reduction of the jet power $N = G_N/\alpha$, where α is the specific weight of the power source independent of N, was considered in [1]. In [2] the limiting case of continuous optimal jet power reduction with the corresponding power source weight reduction was considered, and the consideration of multistep power reduction was continued.

Below are determined the optimal parameters for motion of a body of variable mass taking into account the random processes in power reduction of the source.

1. Formulation of the problem. Let as the result of random factors at times t_j the separate elements of the power source go out of action. We will assume that these elements or the complete power source consist of a certain number n of autonomous sections which are deactivated at the time of damage which results, in the general case, in the change of the work regime of the remaining sections and in reduction of the jet power

$$N_j = \varepsilon_j N_0 \qquad (0 \leqslant \varepsilon_j < \varepsilon_{j-1} < \ldots < \varepsilon_0 = 1) \tag{1.1}$$

where N_0 is the initial jet power, ε_j are parameters defined by the type of the power source and by the number of working sections.

The specific weight of the power source, apparently, depends on the number of autonomous sections n which for a fixed value N_0 is bounded from above because of the existence of a certain minimum size of the autonomous section $n \leqslant n_{max} = N_0/\Delta N_{min}$. The weight of the power source during motion is assumed constant: $G_N = \alpha N_0$.

For minimum instantaneous consumption of fuel (working medium) for a given thrust and power source the jet power must be the maximum possible for the condition of the power source at a given time. If this requirement does not result in an increased probability of power source damage then it will also be necessary for the minimum of the required fuel reserve. Confining ourselves to this case, we will assume that the working regime of the undamaged sections is chosen such that the quantity N_j is the maximum possible and that in the areas among the damaged sections the jet power is equal to this quantity $N = N_j$ for $t_j \leqslant t \leqslant t_{j+1}$.

Let the instants of time t_j be known for some realized motion, then applying to equations of [1] to the areas among the damaged sections we obtain the following expression for the weight of the body G_k at the termination of motion ($t = t_k$):

$$\frac{G_k}{G_0} = \left(1 + \frac{G_0}{2N_0 g} \sum_{j=0}^{\nu} \frac{1}{\varepsilon_j} \int_{t_j}^{t_{j+1}} a^2\, dt\right)^{-1} \quad (|\mathbf{a}| = a) \tag{1.2}$$

where G_0 is the initial weight of the body, $\mathbf{a}(t)$ is the thrust acceleration (thrust divided by mass flow of the body), g is the earth gravity acceleration, $< n$ is the number of the instant of time of the last damage (it is assumed conditionally that $t_{\nu=+1} = t_k$).

For the prediction of the motion characteristics we utilize Formula (1.2) in which the times of damage t_j are taken as the average times of damage determined from the condition of reducing the integral for damage probability of the power source into unity per unit time, computed along the trajectory (see further Equations (2.1) and (2.2)). At the same time the probability of simultaneous deactivation of several sections is neglected in the following, i.e. the number of the time instance of the expected damage coincides with the number of the deactivated sections*.

Now, analogous to [1], we will formulate the problem for finding a minimum of the composite relative weight $G^\circ = G_N^\circ + G_M^\circ$ of the power source $G_N^\circ = \alpha N_0/G_0$ and the averaged reserve of fuel $G_M^\circ = 1 - G_k/G_0$, required for carrying out the given maneuver with a known dependence of

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* The so obtained average characteristics will not necessarily coincide with the mathematical expectations of the corresponding quantities. But in the capacity of the first approach for solving the problem the above formulated procedure is utilized.

the power source damage probability per unit time on the number of working sections, the amount of power, body coordinates, and time*.

The quantity G° which can be expressed by the following formula

$$G^{\circ} = \frac{\Phi / G_N^{\circ}}{1 + \Phi / G_N^{\circ}} + G_N^{\circ} \qquad \left(\Phi = \frac{\alpha}{2g} \sum_{j=0}^{\nu} \frac{1}{\varepsilon_j} \int_{t_j}^{t_{j+1}} a^2 \, dt\right) \tag{1.3}$$

decreases monotonically with the decrease of Φ (here and in the following t_j denotes the averaged moments of damage). Therefore the problem can be divided into two parts:

1. For the given quantities N_0, n and for a known dependence of damage probability of the power source on the above indicated parameters, determine the time variation of the thrust acceleration $\mathbf{a}_{ext}(t)$ such that for the displacement between two given points in the phase space

$$\{\mathbf{r}(t_0) = \mathbf{r}_0, \ \dot{\mathbf{r}}(t_0) = \dot{\mathbf{r}}_0\}, \ \{\mathbf{r}(t_k) = \mathbf{r}_k, \ \dot{\mathbf{r}}(t_k) = \dot{\mathbf{r}}_k\}$$

at given instants of time t_0 and t_k the functional Φ have a minimum

$$\Phi \left[\mathbf{a}_{ext}(t)\right] = \Phi_{min}(N_0, n)$$

2. For given quantities G_0 and ΔN_{min}, and also for the known relations $\alpha = \alpha(n)$ and $\Phi = \Phi_{min}(N_0, n)$, determine the optimal values of N_0 and n, which ensure the minimum of the function G°, defined by Formula (1.3). In the following the first of the formulated problems is considered.

2. The equations of the variational problem. Let us consider two types of random processes of power source damage, the character of which substantially affects the equations of the variational problem.

(a) the processes dependent on nonhomogeneous external conditions (the probability of damage depends on $\mathbf{r}$);

(b) internal processes and processes dependent on homogeneous external conditions (the probability of damage is independent of $\mathbf{r}$). Considering the random process of deactivation of a certain section of the power source independent from deactivation of other sections we will write down the probability p of deactivating one working section at time t as follows:

The damage probability can also depend on other parameters defined by the specific type of the random process; their introduction does not involve difficulties.

$$\begin{array}{lll} (\text{а}) & p(\mathbf{r}, t) = p_0(N_0)\left(1 - \dfrac{j}{n}\right)\rho_1(\mathbf{r}, t) & \left(\begin{array}{l} t_j \leqslant t \leqslant t_{j+1} \\ j = 0, 1, \ldots \nu \end{array}\right) \\ (\text{б}) & p(t) = p_0(N_0)\left(1 - \dfrac{j}{n}\right)\rho_2(t) & \end{array} \qquad (2.1)$$

Here $p_0(N_0)\rho_1(\mathbf{r}_0, t_0)$ and $p_0(N_0)\rho_2(t_0)$ are the probabilities of power source damage at the beginning of motion; ρ_1 and ρ_2 are functions defined by the character of the random process and source. The averaged time t_{j+1} is determined from the condition of setting to unity the time integral of the probability $p(t)$ computed along the trajectory $\mathbf{r}(t)$ from the averaged instant t_j of the preceding damage. The time of the last expected damage ($t = t_\nu$) is found from the condition that this integral from t_ν to t_k is less than or equal to unity. For the probability p determined for t_j (2.1) we obtain the following relationships:

$$\begin{array}{lll} (\text{a}) & \displaystyle\int_{t_j}^{t_{j+1}} \rho_1[\mathbf{r}(t), t]\, dt = \frac{1}{p_0(1 - j/n)}, & \displaystyle\int_{t_\nu}^{t_k} \rho_1[\mathbf{r}(t), t]\, dt \leqslant \frac{1}{\rho_0(1 - \nu/n)} \\ (\text{б}) & \varphi(t_{j+1}) - \varphi(t_j) = \dfrac{1}{p_0(1 - j/n)}, & \varphi(t_k) - \varphi(t_\nu) \leqslant \dfrac{1}{p_0(1 - \nu/n)} \end{array}$$

$$\left(j = 0, 1, \ldots, \nu - 1;\ \varphi(t) = \int \rho_2(t)\, dt\right) \qquad (2.2)$$

Eliminating in the expression for Φ (Formula (1.3)) the thrust acceleration $\mathbf{a}$ with the aid of the equations of motion in the gravitational field $\ddot{\mathbf{r}} = \mathbf{a} - \mathbf{R}$, where $\mathbf{R} = \mathbf{R}(\mathbf{r}, t)$ is the acceleration from gravity forces, we transform the functional Φ into

$$\Phi = \frac{\alpha}{2g} \sum_{j=0}^{\nu} \frac{1}{\varepsilon_j} \int_{t_j}^{t_{j+1}} (\ddot{\mathbf{r}} + \mathbf{R}, \ddot{\mathbf{r}} + \mathbf{R})\, dt \qquad (2.3)$$

For the minimum of the functional Φ it is necessary that the trajectory consist from the extremals of the functionals

$$\begin{array}{ll} (\text{a}) & I_j = \dfrac{1}{\varepsilon_j} \displaystyle\int_{t_j}^{t_{j+1}} [(\ddot{\mathbf{r}} + \mathbf{R}, \ddot{\mathbf{r}} + \mathbf{R}) + \lambda_j \rho_1(\mathbf{r}, t)]\, dt \\ (\text{б}) & I_j = \dfrac{1}{\varepsilon_j} \displaystyle\int_{t_j}^{t_{j+1}} (\ddot{\mathbf{r}} + \mathbf{R}, \ddot{\mathbf{r}} + \mathbf{R})\, dt \end{array} \qquad (2.4)$$

where $j = 0, 1, \ldots, \nu$; λ_j are constants with the condition of isoperimetry (first two equations of (2.2)); i.e. in the inertial rectangular coordinate system $\{r^{(1)}, r^{(2)}, r^{(3)}\}$ the following Euler equations should be fulfilled along the trajectory:

$$\text{(a)} \qquad \ddot{a}^{(i)} + \left(\mathbf{a}, \frac{\partial \mathbf{R}}{\partial r^{(i)}}\right) + \lambda_j \frac{\partial \rho_1}{\partial r^{(i)}} = 0 \quad \text{for } t_j \leqslant t \leqslant t_{j+1}$$

$$\text{(b)} \qquad \ddot{a}^{(i)} + \left(\mathbf{a}, \frac{\partial \mathbf{R}}{\partial r^{(i)}}\right) = 0 \; (i = 1, 2, 3; \; j = 0, 1, \ldots, \nu; \; \lambda_\nu = 0) \qquad (2.5)$$

From the first equation of (2.5) it can be seen that with the space nonhomogeneous external conditions (derivative $\partial \rho_1 / \partial r^{(i)} \neq 0$) the dimensionality of the extremal trajectory can increase, i.e. the one-dimensional or two-dimensional damage-free trajectory can become two or three-dimensional, respectively. The second equation (2.5), as was shown in [1], permits two first integrals in the case of a central gravitational field, while in the case of a homogeneous gravitational field it has an analytical solution $\mathbf{a}(t) = \mathbf{c}_1 + \mathbf{c}_2 t$.

Equating the sum of the coefficients for variations of the radius vector and velocity for $t = t_j$ in the expression for first variation of the functional Φ on the extremals, we obtain the following conditions for acceleration and its derivative:

$$\frac{1}{\varepsilon_j} \mathbf{a}_j^+ = \frac{1}{\varepsilon_{j-1}} \mathbf{a}_j^-, \qquad \frac{1}{\varepsilon_j} \dot{\mathbf{a}}_j^+ = \frac{1}{\varepsilon_{j-1}} \dot{\mathbf{a}}_j^- \qquad (j = 1, 2, \ldots, \nu) \qquad (2.6)$$

Here

$$\mathbf{a}_j^+ = \lim_{\delta \to 0} \mathbf{a}(t_j + \delta), \qquad \mathbf{a}_j^- = \lim_{\delta \to 0} \mathbf{a}(t_j - \delta)$$
$$\dot{\mathbf{a}}_j^+ = \lim_{\delta \to 0} \dot{\mathbf{a}}(t_j + \delta), \qquad \dot{\mathbf{a}}_j^- = \lim_{\delta \to 0} \dot{\mathbf{a}}(t_j - \delta) \qquad (\delta > 0)$$

Thus, in damaging the power source at time $t = t_j$ the magnitude of the acceleration $\mathbf{a}$ and the magnitude of its time derivative must have a discontinuity while both these quantities decrease in passing through the discontinuity at the $\varepsilon_j / \varepsilon_{j-1}$ time:

In case of (a), when the damage probability depends on the trajectory, when forming the first variation of the functional Φ one must also take the variation of the times t_j. This furnishes the additional conditions

$$\rho_1(t_j) \left(\frac{\lambda_j}{\varepsilon_j} - \frac{\lambda_{j-1}}{\varepsilon_{j-1}}\right) + \frac{\varepsilon_{j-1} - \varepsilon_j}{\varepsilon_j^2} (\mathbf{a}_j^+, \mathbf{a}_j^+) = 0 \qquad (j = 1, 2, \ldots, \nu - 1) \qquad (2.7)$$

Thereafter the problem is reduced to finding a solution of the equations of motion coincident with the system (2.5) satisfying the boundary conditions and conditions (2.2) and (2.6), while for case (a), also the conditions (2.7) with the inequalities in (2.2) defining ν.

3. The limiting case*. With a large number of power source sections the solution of the variational problem in the exact formulation is difficult. But if the number of sections is sufficiently large so that $n - \nu \gg 1$, and if the sections are fully autonomous, i.e. $\varepsilon_j = 1 - j/n$, then the stepwise reduction of power

$$N(t) = N_0 (1 - j/n) \quad \text{for } t_j \leqslant t \leqslant t_{j+1} \qquad (j = 0, 1, \ldots, \nu) \tag{3.1}$$

where t_j is defined by Formulas (2.2), can be approximated by a continuous one

$$\dot{N} = -\frac{1}{n} p_0(N_0) \rho(\mathbf{r}, t) N \tag{3.2}$$

Indeed, integrating Equation (3.2) in the range t_j to t_{j+1} we are convinced that $N(t_j) - N(t_{j+1}) = N_0/n$, with accuracy up to terms of order $1/(n - j)^2$, i.e. the power reduction from one moment of damage to another in the limiting case and the exact formulation is identical.

The problem considered in the preceding section is reducible to finding the extremals of the functional

$$\Phi = \frac{\alpha}{2g} \int_{t_0}^{t_k} \left\{ \frac{1}{\varepsilon} (\ddot{\mathbf{r}} + \mathbf{R}, \ddot{\mathbf{r}} + \mathbf{R}) + \sigma(t) \left[\dot{\varepsilon} + \frac{1}{n} p(\mathbf{r}, t) \varepsilon \right] \right\} dt$$
$$\varepsilon = N / N_0, \qquad p(\mathbf{r}, t) = p_0(N_0) \rho(\mathbf{r}, t) \tag{3.3}$$

where $\sigma(t)$ is a variable Lagrange multiplier of the differential constraint (3.2).

The differential equations for the extremals of such a functional in the inertial rectangular coordinate system are of the following form:

$$\ddot{a}^{(i)} + \frac{2}{n} p \dot{a}^{(i)} + \frac{1}{n} \dot{p} a^{(i)} + \left(\mathbf{a}, \frac{\partial \mathbf{R}}{\partial r^{(i)}} \right) + \frac{1}{2n} \sigma \varepsilon^2 \frac{\partial p}{\partial r^{(i)}} = 0 \qquad (i = 1, 2, 3)$$
$$\dot{\sigma} + \frac{1}{\varepsilon^2} (\mathbf{a}, \mathbf{a}) - \frac{1}{n} \sigma p = 0, \qquad \dot{\varepsilon} + \frac{1}{n} p \varepsilon = 0 \tag{3.4}$$

Thus, in place of the conditions (2.2), (2.6) and (2.7) of the exact formulation we have in the limiting case two differential equations (last two equations in the system (3.4)). The following examples show the essential simplification of the solution for the limiting case compared to that of the exact formulation.

4. Examples*. As an example let us consider the case when the power

* For the limiting case the division of the damage processes into (a) and (b) types is inessential in deriving the equations of motion, therefore the equations are derived for the more general case (a).

source consists of fully autonomous sections ($\varepsilon_j = 1 - j/n$), the random damaging process of the power source elements is of type (b), and $\rho_2(t) \equiv 1$, while the motion takes place along a straight line in a force-free field ($\mathbf{R}(\mathbf{r}, t) \equiv 0$), subject to the following boundary conditions:

$$r(t_0) = \dot{r}(t_0) = 0, \qquad r(t_k) = L, \qquad \dot{r}(t_k) = 0, \qquad t_0 = 0, \qquad t_k = T \tag{4.1}$$

Exact formulation. Integrating the second equation in (2.5), taking into account the second condition in (2.2) and (2.6), we obtain

$$\dot{a}_j^+ = \left(1 - \frac{j}{n}\right)\dot{a}_0^+, \qquad a_j^+ = \left(1 - \frac{j}{n}\right)(a_0^+ + \dot{a}_0^+ t_j), \qquad t_j = \frac{1}{p_0}\sum_{i=0}^{j-1}\frac{1}{1 - i/n}$$

$$\dot{r}_j = \frac{1}{p_0}\left[ja_0^+ + \dot{a}_0^+\left(\sum_{i=1}^{j-1} t_i + \frac{1}{2}t_j\right)\right] \tag{4.2}$$

$$r_j = \frac{1}{p_0^2}\left[a_0^+\sum_{i=0}^{j-1}\frac{i + 1/2}{1 - i/n} + \dot{a}_0^+\left(\sum_{i=1}^{j-1}\frac{1}{1 - i/n}\sum_{l=1}^{i} t_l + \frac{1}{6p_0}\sum_{i=0}^{j-1}\frac{1}{(1 - i/n)^2}\right)\right]$$

The quantities a_0^+ and $\dot{a}_0^+$ are determined by solving the system of linear algebraic equations obtained from the boundary conditions

$$a_0^+\left[\frac{\nu}{p_0} + \left(1 - \frac{\nu}{n}\right)(T - t_\nu)\right] + \dot{a}_0^+\left[\frac{1}{p_0}\left(\sum_{j=1}^{\nu-1} t_j + \frac{1}{2}t_\nu\right) + \right.$$
$$\left. + \frac{1}{2}\left(1 - \frac{\nu}{n}\right)(T^2 - t_\nu^2)\right] = 0 \tag{4.3}$$

$$a_0^+\left\{\frac{1}{p_0^2}\sum_{j=0}^{\nu-1}\frac{j + 1/2}{1 - j/n} + (T - t_\nu)\left[\frac{\nu}{p_0} + \frac{1}{2}\left(1 - \frac{\nu}{n}\right)(T - t_\nu)\right]\right\} + \tag{4.4}$$
$$+ \dot{a}_0^+\left\{\frac{1}{p_0^2}\left(\sum_{j=1}^{\nu-1}\frac{1}{1 - j/n}\sum_{i=1}^{j} t_i + \frac{1}{6p_0}\sum_{j=0}^{\nu-1}\frac{1}{(1 - j/n)^2}\right) + \right.$$
$$\left. + (T - t_\nu)\left[\frac{1}{p_0}\left(\sum_{j=1}^{\nu-1} t_j + \frac{1}{2}t_\nu\right) + \frac{1}{6}\left(1 - \frac{\nu}{n}\right)(T - t_\nu)(T + 2t_\nu)\right]\right\} = L$$

while the number of damages for the total time of motion ν is determined according to the second condition of (2.2) from the inequality

$$\frac{1}{p_0}\sum_{j=0}^{\nu-1}\frac{1}{1 - j/n} \leqslant T \leqslant \frac{1}{p_0}\sum_{j=0}^{\nu}\frac{1}{1 - j/n}$$

Thereafter, the functional Φ on the extremal trajectory becomes a function of α, p_0, n and the boundary conditions

$$\Phi_{\min} = \frac{\alpha}{2g}\left\{\frac{1}{p_0}\sum_{j=0}^{\nu-1}\left[(a_0^+ + \dot{a}_0^+ t_j)^2 + (a_0^+ + \dot{a}_0^+ t_j)\frac{\dot{a}_0^+}{p_0(1-j/n)} + \frac{(\dot{a}_0^+)^2}{3p_0^2(1-j/n)^2}\right] + \right.$$
$$\left. + \left(1-\frac{\nu}{n}\right)(T-t_\nu)\left[(a_0^+ + \dot{a}_0^+ t_\nu)^2 + (a_0^+ + \dot{a}_0^+ t_\nu)\,\dot{a}_0^+(T-t_\nu) + \frac{1}{3}(\dot{a}_0^+)^2(T-t_\nu)^2\right]\right\} \tag{4.5}$$

Solutions analogous to the above are obtained also for the case of arbitrary motion in a homogeneous gravitational field ($\mathbf{R}(\mathbf{r}, t) \equiv \mathbf{g}_0 \neq 0$).

Limiting case. The extremal law for the time variation of the thrust acceleration $a_{ext}(t)$ in the limiting case for the above motion is of the following form:

$$a_{\text{ext}}(t) = \frac{L}{T^2}\frac{p_*^3}{1-e^{-p_*}-p_*^2/(e^{p_*}-1)}\left[\left(\frac{1}{p_*}-\frac{1}{e^{p_*}-1}\right)-\frac{t}{T}\right]e^{-p_* t/T}\left(p_* = \frac{p_0 T}{n}\right) \tag{4.6}$$

and the minimal value of the functional Φ is

$$\Phi_{\min} = \frac{\alpha L^2}{2gT^3}p_*^3\left(1-e^{-p_*}-\frac{p_*^2}{e^{p_*}-1}\right)^{-1} \tag{4.7}$$

In the limit for $p_* \to 0$, i.e. in the absence of damages, Formulas (4.6) and (4.7) become

$$a_{\text{ext}}(t)\,|_{p_*=0} = \frac{6L}{T^2}\left(1-2\frac{t}{T}\right), \qquad \Phi_{\min}\,|_{p_*=0} = \frac{6\alpha L^2}{gT^3} \tag{4.8}$$

For $T \to \infty$ and fixed value of p_0/n, the quantity $\Phi_{\min}$ decreases monotonically to $(\alpha/2g)L^2(p_0/n)^3$. If, however, the motion in the presence of damage ($p_0/n \neq 0$) follows a linear law $a(t)$, optimal in the absence of damages, then for $T \to \infty$ and fixed p_0/n the value of the functional Φ tends to infinity and there exists an optimal time of motion $T_{opt} \approx 3.52$ (n/p_0), for which Φ has a minimum value approximately equal to 3.46 $(\alpha/2g)L^2(p_0/n)^3$; for $a_{ext}(t)$, optimal for the same value of p_0/n, and for $T = T_{opt}$ the functional $\Phi_{\min} = 1.68\ (\alpha/2g)L^2(p_0/n)^3$.

The solid lines in Fig. 1 show the dependence of

$$\Phi_* = \Phi_{\min} \Big/ \frac{6\alpha L^2}{gT^3} = \Phi_{\min} \Big/ \Phi_{\min}\,|_{p_*=0}$$

on p_* in the exact formulation for n = 1, 2, 5, 10, 20, 50 as well as for the limiting case $n \to \infty$. It can be seen from the figure that starting with n = 10 the exact and the limiting values of the functional Φ_* are sufficiently close. The solid curve $\nu = n$ in Fig. 1 determines the maximum permissible value of p_* for a given value of n.

For comparison, the dotted lines in Fig. 1 show the analogous curves corresponding to the motion in the presence of damages according to the linear law $a(t)$ (dotted line in Fig. 2) which was optimal in the absence of damages.

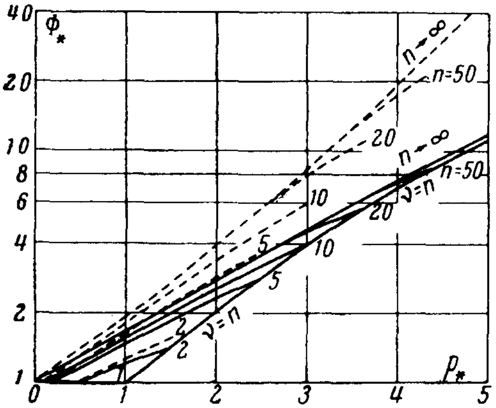

Fig. 1.

The example for the dependence of

$$a_* = \frac{a_{\text{ext}}}{6L/T^2} = \frac{a_{\text{ext}}}{a_{\text{ext}}(0)}\Big|_{p_*=0}$$

on $\theta = \dfrac{t}{T}$

optimal with damages for $n = 2$ and $\rho_* = 3/2$ is shown in Fig. 2 as the solid line.

Motion with coasting. Let there be given the nondimensional time ϑ for operation duration of the power source N:

$$N(t) \equiv a(t) \equiv 0 \quad \text{for} \quad (0 < t_1 \leqslant t \leqslant t_2 < T), \qquad \vartheta = 1 - (t_2 - t_1)/T \leqslant 1$$

Assuming that in turning off the power source one can avoid the damages in its sections let us consider the following function $p(t)$:

$$p(t) = \begin{cases} p_0 \neq 0 & \text{for } 0 \leqslant t \leqslant t_1, \quad t_2 \leqslant t \leqslant T \\ 0 & \text{for } t_1 \leqslant t \leqslant t_2 \end{cases}$$

Applying the equations for the limiting case we obtain the following property of such a motion: if for

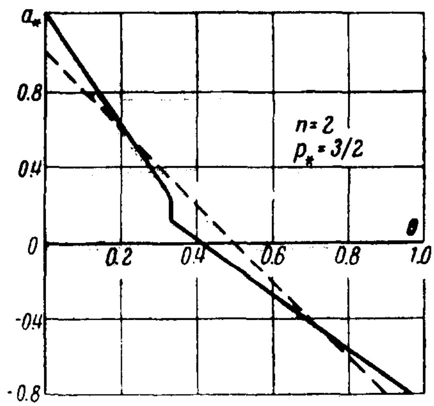

Fig. 2.

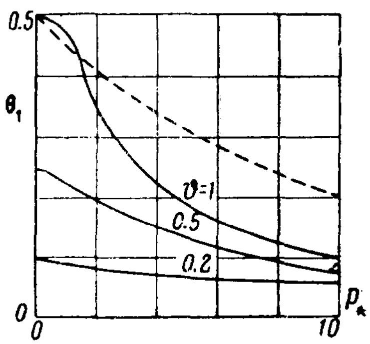

Fig. 3.

$\rho_* = 0$ the optimal value $\theta_1 = t_1/T = \vartheta/2$, i.e. the coasting (passive) region is located symmetrically, then for $\rho_* \neq 0$ the coasting region is shifted in time toward the start of the trajectory (see solid lines in Fig. 3 – the dependence of θ_1 on ρ_* for $\vartheta = 1, 0.5, 0.2$). Despite the fact that $\rho = 0$ for $a = 0$ the functional Φ on the extremal trajectory increases monotonically with decrease in ϑ.

Motion with return. Let

$$r(0) = \dot{r}(0) = 0, \qquad r(t_1) = L, \qquad \dot{r}(t_1) = 0, \qquad r(T) = \dot{r}(T) = 0 \qquad (0 < t_1 < T)$$

For this motion the property similar to that noted for the motion with coasting is characteristic: optimal displacement to $r = L$ occurs faster than the return to $r = 0$ (see dotted curve in Fig. 3 – dependence of θ_1, on ρ_*). In other words: at the initial region of motion, when the power has not yet decreased significantly, it is advantageous to have a larger thrust than at the following region, since it is possible to ensure a sufficiently high velocity of fuel outflow.

BIBLIOGRAPHY

1. Grodzovskii, G.L., Ivanov, Iu.N. and Tokarev, V.V., O dvizhenii tela peremennoi massy s postoiannoi zatrotoi moshchnosti v gravitatsionnom pole (On the motion of a variable mass body with constant power use in a gravitational field). *Dokl. Akad. Nauk USSR* Vol. 137, No. 5, 1961.

2. Ivanov, Iu.N., Optimal'noe izmenenie moshchnosti pri dvizhenii tela peremennoi massy v gravitatsionnom pole (Optimal variation of power for the motion of a body of variable mass in a gravitational field). *PMM* Vol. 26, No. 4, 1962.

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